Grade 4 Mathematics, Quarter 3, Unit 3.1

Exploring Properties of 2-Dimensional Figures

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Identify and draw points, lines, line segments, rays, angles (right, acute, obtuse), parallel and perpendicular lines.
- Classify two-dimensional figures based on presence or absence of parallel and perpendicular lines, and angles of a specified size.
- Identify right triangles and recognize right triangles as a category.
- Generate a shape pattern that follows a given rule and identify apparent features of the pattern that were not explicit in the rule itself.
- Measure angles in whole-number degrees using a protractor.
- Recognize that angle measure is additive.
- Recognize that a measure of an angle is composed of *n* turns of one-degree; where the number of one-degree turns equals the measure of the angle.

Mathematical practices to be integrated

Use appropriate tools strategically.

- Decide when tools are appropriate and helpful.
- Use tools and reasonability to deepen understanding.

Model with mathematics.

- Simplify the situation to a simpler rule allowing for revisions.
- Solve problems in everyday life through the use of algorithms, geometry, a function, etc.

Look for and make use of structure.

- Look for patterns to express regularity in repeated reasoning.

Essential questions

- Given attributes (points, lines, line segments, rays, angles (right, acute, obtuse), parallel and perpendicular lines, how can you draw a figure that represents these attributes?
- Given a figure, how can you identify points, lines, line segments, rays, angles (right, acute, obtuse), parallel and perpendicular lines?
- How can you classify a two-dimensional figure based on the absence or presence of parallel and perpendicular lines, and angles?
- What is a right triangle?
- Given several two-dimensional figures, how can you classify a right triangle?
- How can you turn a rule into a pattern?
- Given a pattern, what are all the features you see?
- What does the measurement of an angle show and how would you measure it?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry 4.G

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

Measurement and Data 4.MD

Geometric measurement: understand concepts of angle and measure angles.

4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.

b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Operations and Algebraic Thinking 4.OA

Generate and analyze patterns.

4.OA.5 Generate a ~~number or~~ shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. ~~*For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*~~

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In third grade, reasoning with shapes and their attributes is a supporting cluster according to the PARCC Model Content Frameworks. Students used area as an attribute of plane figures and measured areas of figures by counting unit squares (square centimeters, square inches, square feet). They learned that shapes fit into different categories (e.g., rhombi, rectangles, and others). They shared attributes (e.g., having four sides), and recognized that these shapes can be part of a larger category. They learned about quadrilaterals and drew examples of them. Students compared and classified shapes according to the property of shared attributes.

Current Learning

In grade 4, students develop an understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures and symmetry. Students draw points, lines, line segments, rays, angles (acute, obtuse, right), and perpendicular and parallel lines at the developmental level. Additionally, students identify these properties in two-dimensional figures. Furthermore, students classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size at the developmental level. They recognize right angles as a category and identify right triangles.

In relation to angle measure, students learn that angles are measured in reference to a circle with its center the common endpoint of the rays. They also use a protractor to measure and sketch angles of a specified degree. Students discover that angle measure is additive. Finally, students solve addition and subtraction problems to find and unknown angle on a diagram.

At a developmental level, students generate a shape pattern that follows a given rule. They identify apparent features of the pattern that were not explicit in the rule itself. According to the CCSS, understanding that geometric figures can be analyzed and classified, and understanding concepts of angles and angle measure is the third critical area. The PARCC document identifies these areas as additional clusters.

Future Learning

In fifth grade, students will classify two-dimensional figures into categories based on their properties. They will classify two-dimensional figures in a hierarchy based on properties. They will understand concepts of volume and will relate volume to multiplication and addition. They will learn to recognize volume as an attribute of a solid figure and understand concepts of volume measurement. In eighth grade, geometry will again become a major cluster. Students will apply angle measure.

Additional Findings

According to *Principles and Standards for School Mathematics*, "As students move from grade 3 to 5, they should develop clarity and precision in describing the properties of geometric objects and then classifying them by these properties into categories such as rectangle, triangle, pyramid, or prism. They can develop knowledge about how geometric shapes are related to one another and begin to articulate geometric arguments about the properties of these shapes" (p. 165).

The book also states, "As students sort, build, draw, model, trace, measure and construct, their capacity to visualize geometric relationships will develop. At the same time they are learning to reason and to make, test, and justify conjectures about these relationships" (p. 165).

Students must understand concepts of angle measurement and presumably gain some experience measuring angles. Before that can happen, students must have some familiarity with geometric terms that are used to define angles as geometric shapes … addition of fractions and concepts of angle measure are connected in that a 1-degree measure is a fraction of an entire rotation and that adding angle measures together is adding fractions with a denominator of 360 (*PARRC Model Content Frameworks*, p. 20).

Grade 4 Mathematics, Quarter 3, Unit 3.2

Applying Multiplication to Fractions

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Solve word problems involving multiplication of a fraction by a whole number
- Use visual models and equations to represent a problem
- Use a visual fraction model to represent *a/b* as the product of *a* x (*1/b*)
- Represent a multiple of *a/b* as a multiple of *1/b*
- Multiply a fraction by a whole number

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Plan a solution pathway.
- Explain the meaning of the problem to themselves.
- Analyze relationships.

Reason abstractly and quantitatively.

- Make sense of quantities in problems**.**
- Attend to the meaning of quantities**.**

Model with Mathematics

- Use visual models to represent the problem.
- Identify important quantities and express them as an equation.

Essential questions

- How do you use a visual model to represent multiplication of a fraction by a whole number?
- How can you use a unit fraction and a whole number to represent 5/4?
- Given a real world problem involving multiplication of a fraction by a whole number, how can you represent the problem and find the solution?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions[3] 4.NF

[3] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction *a/b* as a multiple of *1/b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*

b. Understand a multiple of *a/b* as a multiple of *1/b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

In third grade, students developed understanding of fractions as numbers. They learned part to whole relationships as the quantity formed by 1 part when a whole is partitioned into equal parts. Additionally, students learned to understand and represent fractions as a number on a number line. Finally, they explained equivalence of fractions in special cases by using a visual models and compared fractions by reasoning about their size. They were also able to recognize that comparisons are valid only when two fractions refer to the same whole, and recorded the results to comparisons using the symbols >, <, =, justifying their conclusions.

Current Learning

In grade 4, the development of understanding of fraction equivalence and operations of fractions is a critical area of instruction according to the CCSS. According to the PARCC Model Content Frameworks, building fractions from unit fractions by applying and extending previous understandings of operations on whole numbers is a major cluster. Students understand a fraction a/b as a multiple of 1/b (e.g., 5/4 is the product of 5 x (1/4)). Furthermore, students understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number (Ex. 3 x (2/5) as 6 x (1/5) using visual models as 6/5 recognizing the product). Finally, students solve word problems involving multiplication of a fraction by a whole number. In a previous unit, students explore fractional

equivalence and ordering fractions, as well as, adding and subtracting fractions. **Fourth grade is the first place that students will be working with fractions of a set.**

Future Learning

In fifth grade, students will apply and extend previous understandings of multiplication and division to multiply and divide fractions. They will interpret a fraction as division of the numerator by the denominator. Also, they will solve word problems involving the division of whole numbers leading to answers in the form of fractions or mixed numbers.

Additional Findings

Students apply their understanding of multiplication of whole numbers to fractions. They see a fraction as a numerator times a unit fraction with the same denominator. Additionally, students solve word problems involving multiplication of a fraction by a whole number. (*Progression: 3-5 Number and Operations – Fractions*, p. 7-8)

In grade 4, knowledge of fractions is extended to include wholes that are collections of objects (*Progression: 3-5 Number and Operations – Fractions*, p. 2).

According to the *PARCC Content Model Frameworks*, students extend their prior understanding of multiplication to multiply a fraction by a whole number (p. 21).

Grade 4 Mathematics, Quarter 3, Unit 3.3

Understanding the Relationship Between Fractions and Decimals

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Use decimals to hundredths within the context of money.
- Use decimals to tenths within the context of metric measurement.
- Use models, number lines, or explanations to identify, compare, and order decimals.
- Use the four operations to solve word problems involving money.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities in problem situations**.**
- Attend to meanings of quantities**.**
- Know and flexibly use different properties of operations**.**

Construct viable arguments and critique the reasoning of others.

- Justify conclusions**.**
- Clarify thinking through reasoning**.**

Model with mathematics.

- Use visual models to represent the problem**.**
- Identify important quantities and express them as an equation**.**

Essential questions

- How are decimals and fractions related?
- How can you use decimal notation to represent a fraction? *(denominators of 10 and 100 only)*
- How can you compare or order decimals using a number line or model?
- How can you use a model to identify a decimal?
- How can you use decimals to show amounts of money?
- What strategy do you use for adding or subtracting money?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions[3] 4.NF

[3] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Understand decimal notation for fractions, and compare decimal fractions.

4.NF.6 Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

Measurement and Data 4.MD

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.2 Use the four operations to solve word problems involving ~~distances, intervals of time, liquid volumes, masses of objects, and~~ money, including problems involving simple fractions or decimals, ~~and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.~~

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

Students in third grade gained a firm understanding of fractions as a part of whole and how to represent numbers on a number line. (Denominators limited to 2,3,4,6, and 8) They also learned to solve two-stop word problems using the four operations.

Current Learning

According the PARCC Model Content Frameworks in Mathematics, these standards are a major cluster of study. The CCSS lists work with fractions as one of the three critical areas for study. Students develop their understanding of concepts of fraction equivalence and elements of fraction arithmetic in previous units this year. This understanding is imperative for students to work on concepts in this unit. In fourth grade, students use decimal notation for fractions with denominators of 10 or 100. Students compare two decimals to hundredths by reasoning about size. They record the results of comparisons with the symbols <, =, or >, and justify the conclusions. Students use the four operations to solve word problems involving money.

Future Learning

Students will learn how to solve problems involving multiplication of fractions and mixed numbers. They will be using visual fraction models or equations to represent problems. Students will also be applying and extending previous understandings of division to divide unit fractions by interpreting division of a unit fraction by a non-zero whole number and compute quotients. They will interpret division of a whole number by a unit fraction and compute quotients. They will also solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions.

Additional Findings

Fractions with denominators of 10 and 100, called decimal fractions, arise naturally when students convert between dollars and cents, and have a more fundamental importance developing in grade 5, in the base 10 system. For example, because there are 10 dimes in a dollar, 3 dimes in 3/10 of a dollar; and it is also 30/100 of a dollar because it is 30 cents, and there are 100 cents in a dollar. - PARCC